Kalman Filter Tracking with Geant4e

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Generic Kalman Filter Tracking

- GenFit+Geant4e
  - GenFit: math part, averaging the measurement and predictions.
  - Geant4e: error propagator, make predictions

- Pros:
  - Compatible with Geant4 Geometry
  - General purposed, fits various detector layouts
- Cons: (Geant4e)
  - Hard to run Geant4 and Geant4e at the same time.
  - Some difficulties mentioned in Philipp Oehler's thesis: http://ekp-invenio.physik.uni-karlsruhe.de/record/48184/files/iekp-ka2012-18.pdf

- Luckily, Kun Liu(LANL) already made it work for E906.
  - https://github.com/liuk/kTracker
  - They got much improved Jpsi/psi' peak using this method.
  - I am starting from his work.

Geant4e

- Detailed introduction to Geant4e:
  - GEANT4E talk, Pedro Arce: https://geant4.web.cern.ch/geant4/results/talks/CHEP06/CHEP06-GEANT4E.pdf
  - Geant4e already included in Geant4 already.

- Initialized with a Detector setup.
- G4eTrajState (particle type, position, momentum, errors)
- G4eTarget
  - Surface: G4eTargetPlaneSurface, G4eTargetCylindrycalSurface, etc.
  - Length: Stop until a certain tracking length
  - Volume: Track is propagated until the surface of a GEANT4 volume

Kun’s kTracker

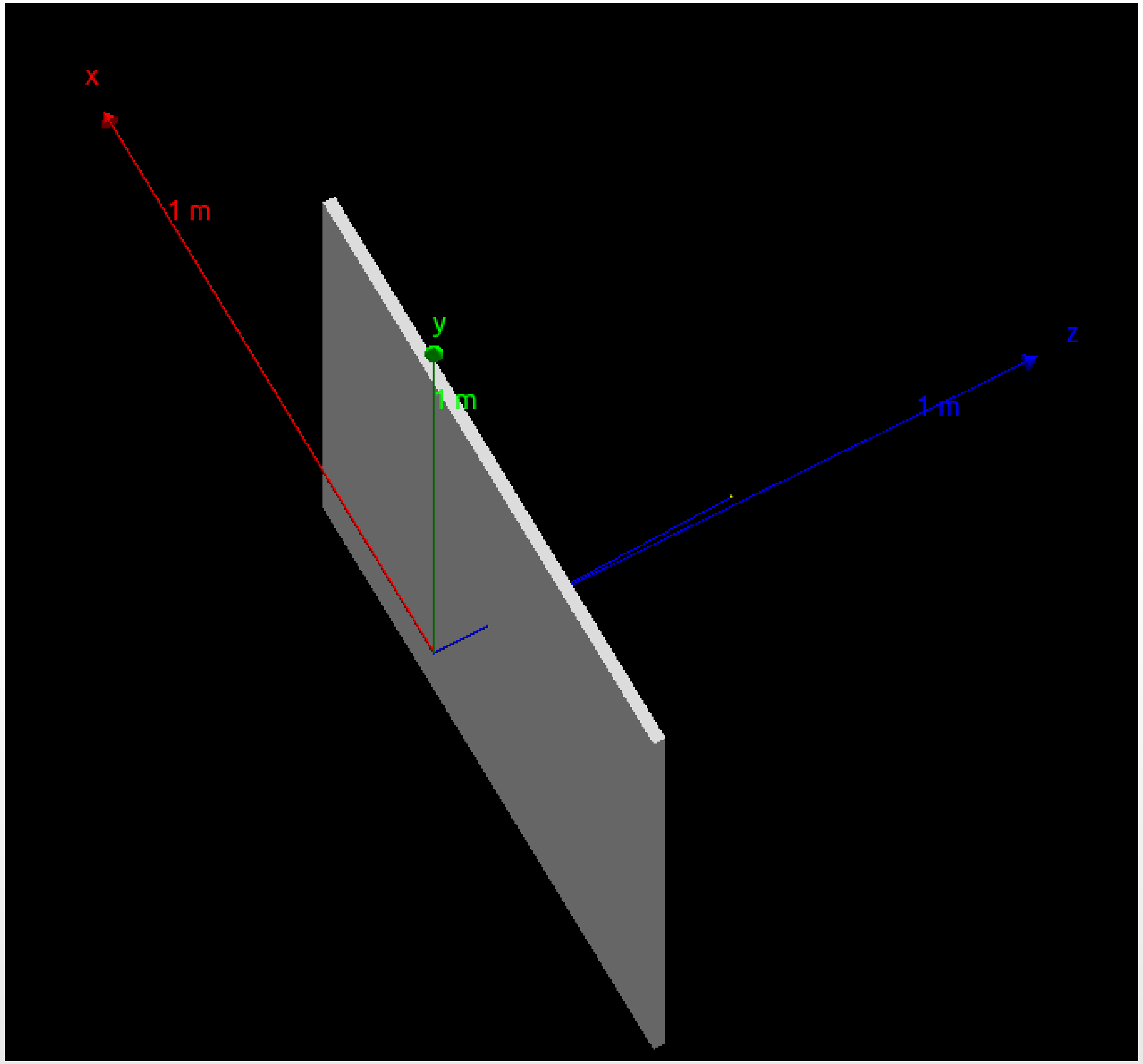
- Designed for E906’s z plane detector layout, means using “G4eTargetPlaneSurface” as the target.
- Coordinate frame transformation in Kun’s kTracker:
  - Geant4e (at least in Geant4.9.6) handle curvilinear coordinate systems well. (Information from Kun)
  - Need transform from detector system to curvilinear system
  - Some math on the transformation: http://www.sciencedirect.com/science/article/pii/S0168900206013143

Today's Topic: Testing basic functions of Geant4e with Kun's kTracker

Geant4e version:
- Geant4.10.1-p02

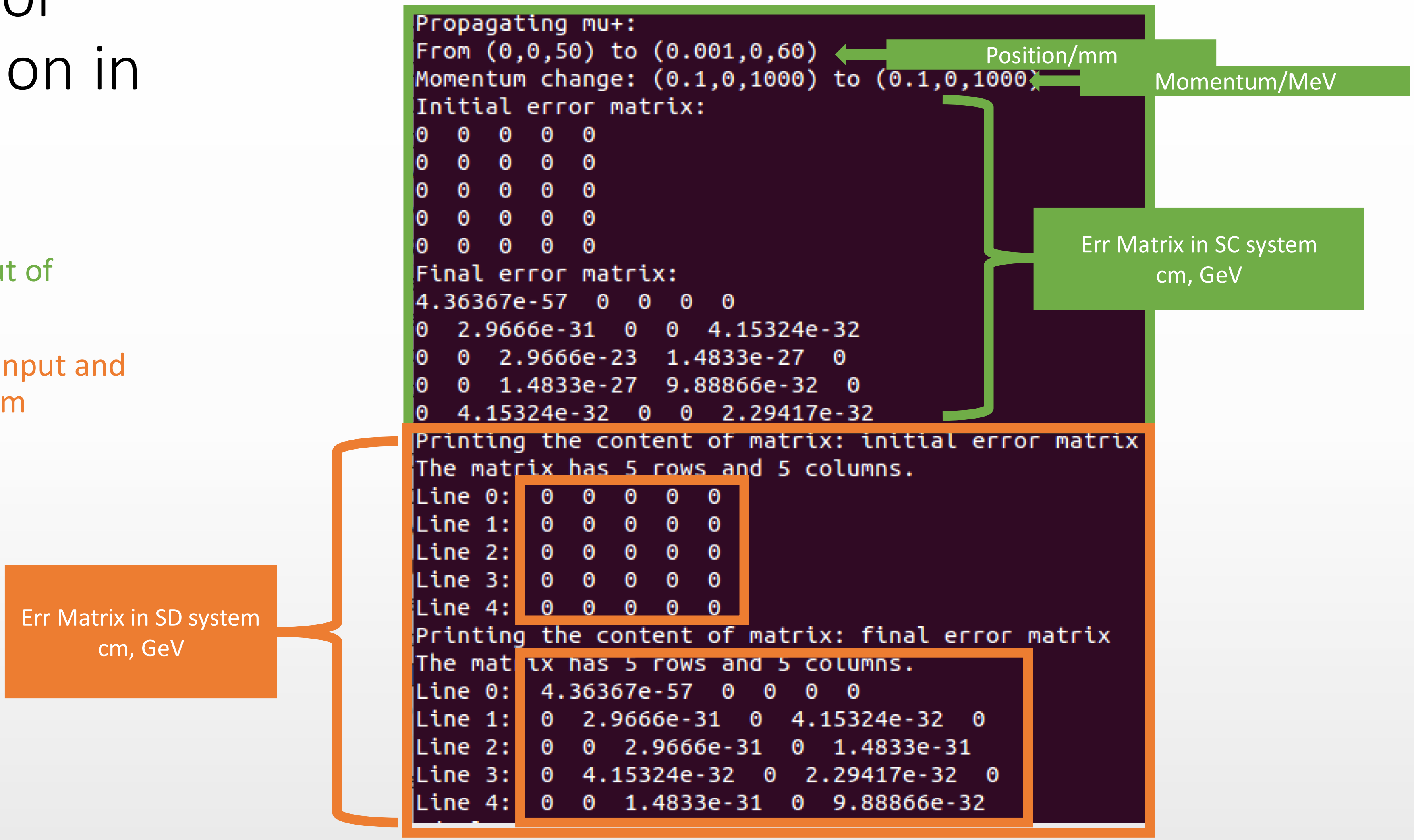
Detector Setup:
- G4_Galactic in the World.
- A Pb/Al sheet perpendicular to z axis. (9-11cm)
- Magnetic field along x axis. (Mostly turned off)
- μ+ injected from (0,0,0) with momentum(1MeV, 0, 10GeV)
- * error if set px to 0.

Track State vector: (1/p, px/pz, py/pz, x, y)
- z used as the propagation position indicator.
- Units: cm, GeV
- In the TrackExtrapolator::print(): mm, MeV



Scenario 1: 0 Error Matrix Propagation in Vacuum

- 0 Error Matrix remained 0.
- Green box indicate the output of TrackExtrpolator::print()
- Yellow box indicates the the input and output err. matrix in SD system



```
Propagating mu+:
From (0,0,50) to (0.001,0,60)
Momentum change: (0.1,0,1000) to (0.1,0,1000)
Initial error matrix:
0   0   0   0   0
0   0   0   0   0
0   0   0   0   0
0   0   0   0   0
0   0   0   0   0
Final error matrix:
4.36367e-57   0   0   0   0
0   2.9666e-31   0   0   4.15324e-32
0   0   2.9666e-23   1.4833e-27   0
0   0   1.4833e-27   9.88866e-32   0
0   4.15324e-32   0   0   2.29417e-32
Printing the content of matrix: initial error matrix
The matrix has 5 rows and 5 columns.
Line 0:  0   0   0   0   0
Line 1:  0   0   0   0   0
Line 2:  0   0   0   0   0
Line 3:  0   0   0   0   0
Line 4:  0   0   0   0   0
Printing the content of matrix: final error matrix
The matrix has 5 rows and 5 columns.
Line 0:  4.36367e-57   0   0   0   0
Line 1:  0   2.9666e-31   0   4.15324e-32   0
Line 2:  0   0   2.9666e-31   0   1.4833e-31
Line 3:  0   4.15324e-32   0   2.29417e-32   0
Line 4:  0   0   1.4833e-31   0   9.88866e-32
```

Scenario 2: Momentum Error Propagation in Vacuum

Momentum uncertainty propagated to position uncertainty properly.

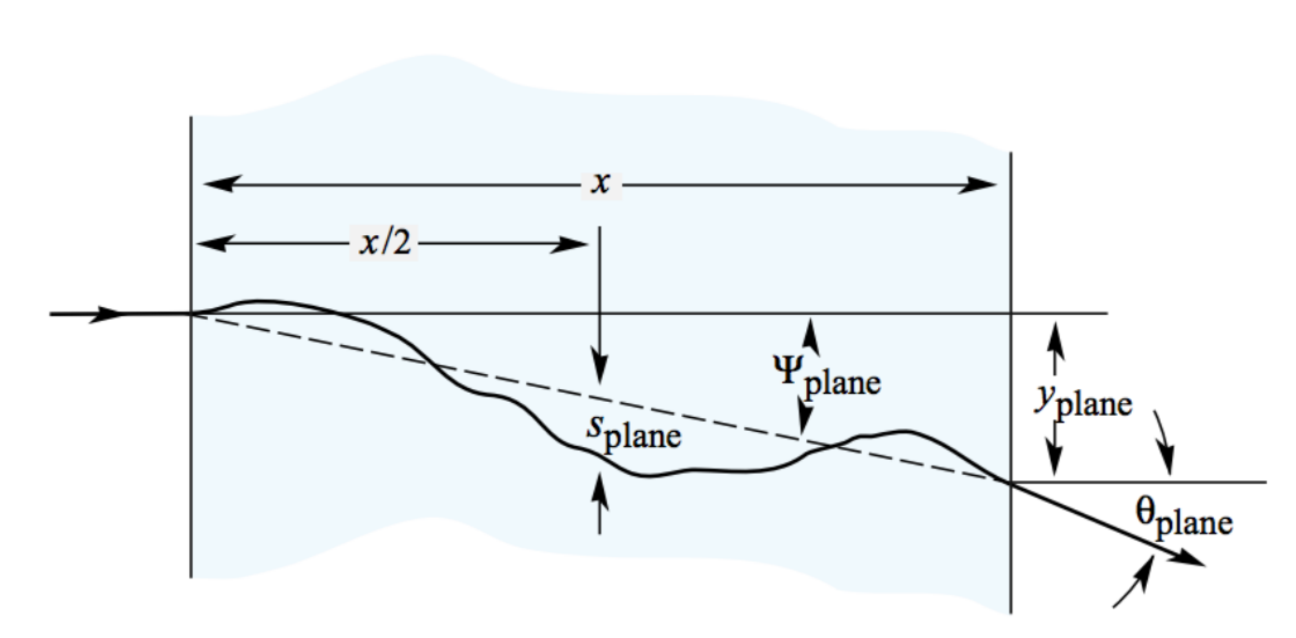
```
Propagating mu+:
From (0,0,50) to (0.001,0,60)
Momentum change: (0.1,0,1000) to (0.1,0,1000)
Initial error matrix:
0  0  0  0  0
0  0  0  0  0
0  0  1e+07  0  0
0  0  0  0  0
0  0  0  0  0
Final error matrix:
4.36367e-57  0  0  0  0
0  2.9666e-31  0  0  4.15324e-32
0  0  1e+07  1000  0
0  0  1000  0.1  0
0  4.15324e-32  0  0  2.29417e-32
Printing the content of matrix: initial error matrix
The matrix has 5 rows and 5 columns.
Line 0:  0  0  0  0  0
Line 1:  0  0  0  0  0
Line 2:  0  0  0.1  0  0
Line 3:  0  0  0  0  0
Line 4:  0  0  0  0  0
Printing the content of matrix: final error matrix
The matrix has 5 rows and 5 columns.
Line 0:  4.36367e-57  0  0  0  0
Line 1:  0  2.9666e-31  0  4.15324e-32  0
Line 2:  0  0  0.1  0  0.1
Line 3:  0  4.15324e-32  0  2.29417e-32  0
Line 4:  0  0  0.1  0  0.1
```

Scenario 3: Multiple Scattering through Material

http://pdg.lbl.gov/2015/reviews/rpp2015-rev-passage-particles-matter.pdf

$$\theta_0 = \frac{13.6\ \mathrm{MeV}}{\beta c p}\, z\, \sqrt{x/X_0}\Big[1 + 0.038\ln(x/X_0)\Big]$$

$$y^{\mathrm{rms}}_{\mathrm{plane}} = \frac{1}{\sqrt{3}}\, x\, \theta^{\mathrm{rms}}_{\mathrm{plane}} = \frac{1}{\sqrt{3}}\, x\, \theta_0\ ,$$



Scenario 3: Multiple Scattering through Material

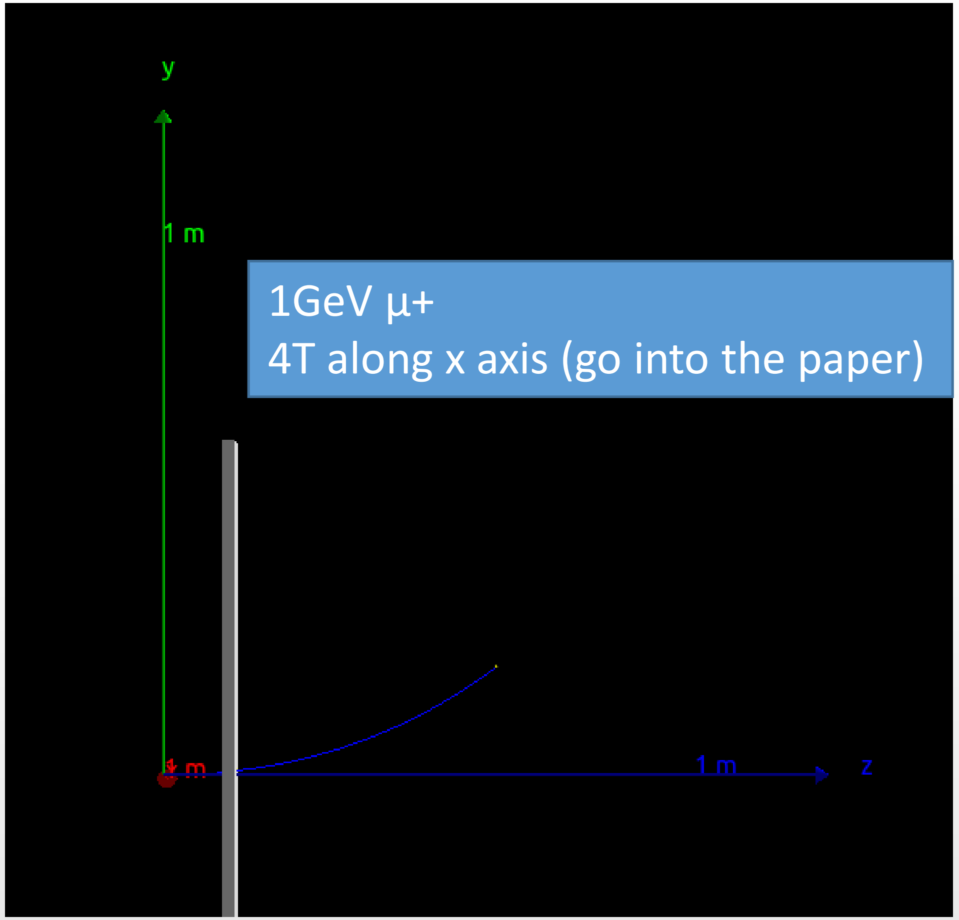
2cm Pb, 10 GeV mu+	Calculation	Geant4e
θ0	0.0027	0.0026
y0 / cm	0.0031	0.0030

2cm Al, 10 GeV mu+	Calculation	Geant4e
θ0	0.00061	0.00065
y0 / cm	0.00070	0.00074

```
Propagating mu+:
From (0,0,90) to (0.002,0,110)
Momentum change: (0.1,0,1000) to (0.0971525,0,971.525)
Initial error matrix:
0  0  0  0  0
0  0  0  0  0
0  0  0  0  0
0  0  0  0  0
0  0  0  0  0
Final error matrix:
2.06132e-05  0  0  0  0
0  0.000690246  0  0  0.000407678
0  0  69024.6  6.83776  0
0  0  6.83776  0.000907419  0
0  0.000407678  0  0  0.000470837
Printing the content of matrix: initial error matrix
The matrix has 5 rows and 5 columns.
Line 0:  0  0  0  0  0
Line 1:  0  0  0  0  0
Line 2:  0  0  0  0  0
Line 3:  0  0  0  0  0
Line 4:  0  0  0  0  0
Printing the content of matrix: final error matrix
The matrix has 5 rows and 5 columns.
Line 0:  2.06132e-05  0  0  0  0
Line 1:  0  0.000690246  0  0.000407678  0
Line 2:  0  0  0.000690246  0  0.000683776
Line 3:  0  0.000407678  0  0.000470837  0
Line 4:  0  0  0.000683776  0  0.000907419
```

2cm Pb

Track propagation in Magnetic Field




```
Prop
From (0,0,50) to (0.001,0.00311784,60)
Momentum change: (1,0,10000) to (1,5.99585,10000)
Initial error matrix:
0  0  0  0  0
0  0  0  0  0
0  0  0  0  0
0  0  0  0  0
0  0  0  0  0
Final error matrix:
4.17651e-61  -8.81481e-64  4.63035e-61  4.53756e-65  -1.76296e-64
-8.81481e-64  2.93904e-33  -2.46272e-31  -1.71507e-34  4.40953e-34
4.63035e-61  -2.46272e-31  2.3839e-26  5.37648e-30  -5.07014e-31
4.53756e-65  -1.71507e-34  5.37648e-30  1.31025e-33  -1.62453e-34
-1.76296e-64  4.40953e-34  -5.07014e-31  -1.62453e-34  2.58265e-34
Printing the content of matrix: initial error matrix
The matrix has 5 rows and 5 columns.
Line 0:  0  0  0  0  0
Line 1:  0  0  0  0  0
Line 2:  0  0  0  0  0
Line 3:  0  0  0  0  0
Line 4:  0  0  0  0  0
Printing the content of matrix: final error matrix
The matrix has 5 rows and 5 columns.
Line 0:  4.17651e-61  -1.32616e-64  9.15775e-64  -1.5755e-65  1.81359e-64
Line 1:  -1.32616e-64  8.60114e-33  -1.09404e-33  3.11383e-33  -7.53984e-34
Line 2:  9.15775e-64  -1.09404e-33  3.14647e-33  -6.17295e-34  5.95309e-34
Line 3:  -1.5755e-65  3.11383e-33  -6.17295e-34  1.22906e-33  -3.24364e-34
Line 4:  1.81359e-64  -7.53984e-34  5.95309e-34  -3.24364e-34  3.39457e-34
```

10GeV μ+
2T along x axis

Summary

- Successful experience from E096, Kun’s work
- Geant4e w/ kTracker basic functions tested.

- To-do:
  - Combine Geant4 with GenFit.
  - Cylindrical frame Jacobean transformations maybe needed.

Useful Links

- GEANT4E talk, Pedro Arce
  - https://geant4.web.cern.ch/geant4/results/talks/CHEP06/CHEP06-GEANT4E.pdf
- CMS talk, Thomas Hauth
  - https://indico.cern.ch/event/279530/session/0/contribution/12/attachments/511926/706535/Hauth_2014-03-LLPC.pdf
- Belle II thesis, Philipp Oehler
  - http://ekp-invenio.physik.uni-karlsruhe.de/record/48184/files/iekp-ka2012-18.pdf
- Kun Liu's work for E906:
  - https://github.com/liuk/kTracker

Backups

```
Propagating mu+:
From (0,0,90) to (0.002,0,110)
Momentum change: (1,0,10000) to (0.998868,0,9988.68)
Initial error matrix:
0  0  0  0  0
0  0  0  0  0
0  0  0  0  0
0  0  0  0  0
0  0  0  0  0
Final error matrix:
1.59669e-10  0  0  0  0
0  4.16424e-07  0  0  2.49704e-07
0  0  41.6424  0.00416274  0
0  0  0.00416274  5.54931e-07  0
0  2.49704e-07  0  0  2.8854e-07
Printing the content of matrix: initial error matrix
The matrix has 5 rows and 5 columns.
Line 0:  0  0  0  0  0
Line 1:  0  0  0  0  0
Line 2:  0  0  0  0  0
Line 3:  0  0  0  0  0
Line 4:  0  0  0  0  0
Printing the content of matrix: final error matrix
The matrix has 5 rows and 5 columns.
Line 0:  1.59669e-10  0  0  0  0
Line 1:  0  4.16424e-07  0  2.49704e-07  0
Line 2:  0  0  4.16424e-07  0  4.16274e-07
Line 3:  0  2.49704e-07  0  2.8854e-07  0
Line 4:  0  0  4.16274e-07  0  5.54931e-07
```

2cm Al

GenFit 5-dim.:

- $\mathbf{q/p} = \frac{q}{|p|}$
- $\mathbf{u'} = \frac{\vec{p}\cdot\vec{u}}{\vec{p}\cdot\vec{n}}$
- $\mathbf{v'} = \frac{\vec{p}\cdot\vec{v}}{\vec{p}\cdot\vec{n}}$
- $\mathbf{u} = (\vec{x} - \vec{O}) \cdot \vec{u}$
- $\mathbf{v} = (\vec{x} - \vec{O}) \cdot \vec{v}$

G4e surface state:

- $\mathbf{1/p} = \frac{1}{|p|}$
- $\mathbf{u'} = \frac{\vec{p}\cdot\vec{u}}{\vec{p}\cdot\vec{n}}$
- $\mathbf{v'} = \frac{\vec{p}\cdot\vec{v}}{\vec{p}\cdot\vec{n}}$
- $\mathbf{u} = (\vec{x} - \vec{O}) \cdot \vec{u}$
- $\mathbf{v} = (\vec{x} - \vec{O}) \cdot \vec{v}$

G4e free state [22] [23]:

- $\mathbf{1/p} = \frac{1}{|p|}$
- $\boldsymbol{\lambda} = \pi/2 - \theta$
- $\boldsymbol{\phi} = \mathrm{atan2}(p_x, p_y)$
- $\boldsymbol{y}_{\perp} = -\sin(\phi)\cdot x + \cos(\phi)\cdot y$
- $\boldsymbol{z}_{\perp} = -\sin(\lambda)\cos(\phi)\cdot x - \sin(\lambda)\sin(\phi)\cdot y + \cos(\lambda)\cdot z$

We use in GENFIT the track representation : RKTrackRep

Track representation

- The track is defined in "The Detector System (SD)" : $(q/p, u', v', u, v)$
- (u, v, w) is an orthonormal reference system with the uv plane coincident with the detector one.
- The derivatives u' (slope) , v' (slope) indicate the momentum direction variation in the SD system for the particle of charge q

Track propagation/model

- The track representation RKTrackRep was adopted from the COMPASS experiment and uses a Runge-Kutta solver to follow particles through magnetic fields.
- It also uses the TGeo classes [ROOT] for the geometry interface.
- The Runge Kutta implementation stems from GEANT3 originally (R. Brun, Sergei Gerrassimov@CERN) et al.)